Pre-Calculus CP 1 - Limits "As x approaches a number" Name: KEY

Determining Limits Algebraically:

- Step #1: Substitute the number that x is approaching INTO the equation!
- If substitution yields....
    - A number, then you are done! ☺
    - A number divided by zero will yield a vertical asymptote (we discussed those yesterday)
    - ZERO divided by ZERO then we will use known limits, graphs, or factoring/rationalizing in order to determine the limit.

Lots of Examples: The first four are the easiest type....

$\lim_{x\to 2} 3 = 3$	$\lim_{x\to -4} x = -4$
$\lim_{x\to 2}(4x^2+3) = 4(2)^2+3 = 19$	$\lim_{x\to \pi}(\sin x) = \sin \pi = 0$

Uh oh...

$\lim_{x\to 2}\sqrt{x-5} = \sqrt{2-5} = \sqrt{-3}$ DNE !

$\lim_{x\to 1}\left(\frac{x^2-1}{x-1}\right)= \lim_{x\to 1}\frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} = \lim_{x\to 1} x+1 = 2$

$\lim_{x\to -3}\frac{x^2+x-6}{x+3}= \lim_{x\to -3}\frac{\cancel{(x+3)}(x-2)}{\cancel{x+3}} = \lim_{x\to -3}(x-2) = -5$

$\lim_{x\to 2}\frac{2-x}{x^2-4}= \lim_{x\to 2}\frac{2-x}{(x+2)(x-2)} = \lim_{x\to 2}\frac{-1\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} = \frac{-1}{4}$

$\lim_{x\to 5}\frac{x^2+5x}{x+5}= \lim_{x\to 5}\frac{x\cancel{(x+5)}}{\cancel{x+5}} = 5$

$$\lim_{x\to5}\left(\frac{\frac{1}{5}-\frac{1}{10-x}}{x-5}\right)=\frac{\frac{1}{5}-\frac{1}{5}}{5-5}=\frac{0}{0}$$ hole!

$$\frac{\frac{10-x-5}{5(10-x)}}{x-5}=\frac{\frac{5-x}{5(10-x)}}{x-5}=\frac{-1(\cancel{x-5})}{5(10-x)}\cdot\frac{1}{\cancel{x-5}}=\frac{-1}{5(10-x)}$$

$$\Rightarrow \lim_{x\to5}\frac{-1}{5(10-x)}=\frac{-1}{5(10-5)}=-\frac{1}{25}$$

$$\lim_{x\to0}\frac{(x-7)^2-49}{x}=\frac{(0-7)^2-49}{0}=\frac{0}{0}$$ hole?

$$\frac{x^2-14x+49-49}{x}=\frac{\cancel{x}(x-14)}{\cancel{x}}\Rightarrow\lim_{x\to0}(x-14)=0-14=-14$$

$$f(x)=\begin{cases}2x+1 & x\le2\\ x-3 & x>2\end{cases}$$

$\lim_{x\to0}f(x)=2(0)+1=1$

$\lim_{x\to5}f(x)=5-3=2$

$\lim_{x\to2}f(x)=$ DNE

$$\lim_{x\to 3}\frac{\sqrt{x+6}-3}{x-3} = \frac{\sqrt{9}-3}{3-3} = \frac{0}{0} \text{ hole!}$$

$$\frac{\sqrt{x+6}-3}{x-3}\left(\frac{\sqrt{x+6}+3}{\sqrt{x+6}+3}\right) = \frac{x+6-9}{(x-3)(\sqrt{x+6}+3)}$$

$$\Rightarrow \lim_{x\to 3}\frac{\cancel{x-3}}{\cancel{x-3}(\sqrt{x+6}+3)} = \lim_{x\to 3}\frac{1}{\sqrt{x+6}+3}$$

$$= \frac{1}{3+3} = \frac{1}{6}$$